disc 105

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Math 128a – Week 5 Worksheet
GSI: Izak, (2/17/21)

2.3 Problems

Problem 1. *Derive the error formula for Newton's method:*

$$|p - p_{n+1}| \leq \frac{M}{2|f'(p_n)|}|p - p_n|^2$$

2.5 Problems

Problem 2. *Steffensen's method is applied to a function $g(x)$ using $p_0^{(0)} = 1, p_2^{(0)} = 3$ to obtain $p_0^{(1)} = .75$. What is $p_1^{(0)}$?*

$x := p_1^{(0)}$ Δ^2

2.6 Problems

Problem 3. *Use Horner's method to evaluate $P(x) = 7x^4 - 2x^2 - 5x - 3$ at $x = 1$*

3.1 Problems

Problem 4. *Given $f(x) = x^3 - 4x^2 + 4$, find the Lagrange interpolation polynomial of degree at most three using the nodes $x_0 = -3, x_1 = -1, x_2 = 1, x_3 = 5$*

Problem 5. *Let $x_0 = -1, x_1 = 0, x_2 = 1$, define $f_0(x) = x^2 - 1, f_1(x) = 2x^2 + 3x$, $f_2(x) = -x^2 + 2x$. Evaluate these polynomials at x_i. Uses this to find a polynomial of degree at most 2 such that $g(x_0) = -4, g(x_1) = -1$, and $g(x_2) = 6$ without preforming any tedious computations.'*

1

3)

1	7	0	-2	-5	-3
		7	7	5	0
	7	7	5	0	$\boxed{-3 = P(1)}$

$P(x) = (7x^3 + 7x^2 + 5x)(x-1) - 3$

$(x-1) \overline{)\, 7x^4 - 2x^2 - 5x - 3}$

1) Trick taylor expand @ $x = p_n$

$f(x) = f(p_n) + f'(p_n)(x - p_n) + \frac{f''(\xi)(x-p_n)^2}{2}$

@ $x = p$

$0 = f(p) = f(p_n) + f'(p_n)(p - p_n) + \frac{f''(\xi)(x-p_n)^2}{2}$

$0 = \frac{f(p_n)}{f'(p_n)} + (p - p_n) + \frac{f''(\xi)(x-p_n)^2}{2f'(p_n)}$

$p_{n+1} := p_n - \frac{f(p_n)}{f'(p_n)}$

$0 = p - p_{n+1} + \frac{f''(\xi)(x-p_n)^2}{2f'(p_n)}$

2) $.75 := \Delta^2(p_0^{(0)}, \underset{x}{p_1^{(0)}}, p_2^{(0)})$

$= 1 - \frac{(x-1)^2}{3 - 2x + 1}$

Solve for x, $x = \{0, 1.5\}$

is x 0 or 1.5

$1 \xrightarrow{g(x)} x \xrightarrow{g(x)} 3$

Δ

.75

	x_0	x_1	x_2
f_0	0	-1	0
f_1	-1	0	5
f_2	-3	0	1
g	2	-1	6

g

$g(x) = f_0 + f_1 + f_2$

$g(x) = af_0 + bf_1 + cf_2$

$2 = g(x_0) = 0 \cdot a - 1 \cdot b - 3 \cdot c$

$-1 = -a$

$6 = 3b + c$

$$\begin{pmatrix} 0 & -1 & -3 \\ -1 & 0 & 0 \\ 0 & 5 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

If you are given $x_0, \ldots, x_n$

Get $L_{n,0}, \ldots, L_{n,n}$ these are polynomials of degree n

If $x_i's$ are distinct, they are a basis of polynomials of degree n

If we want a polynomial f that interpolates on x_i then we know that $f(x) = \sum_{k=0}^{n} c_k L_{n,k}(x)$

This is true for any basis. But $L_{n,i}$ are good, because the c_k are very easy to determine.

They are just $c_k = f(x_k)$