

Math 128a - Week 5 Worksheet  
 GSI: Izak, (2/17/21)

2.3 Problems

Problem 1. Derive the error formula for Newton's method:

$$|p - p_{n+1}| \leq \frac{M}{2|f'(p_n)|} |p - p_n|^2$$

2.5 Problems

Problem 2. Steffensen's method is applied to a function  $g(x)$  using  $p_0^{(0)} = 1, p_1^{(0)} = 3$  to obtain  $p_2^{(0)} = .75$ . What is  $p_1^{(0)}$ ?

2.6 Problems

Problem 3. Use Horner's method to evaluate  $P(x) = 7x^4 - 2x^2 - 5x - 3$  at  $x = 1$

3.1 Problems

Problem 4. Given  $f(x) = x^3 - 4x^2 + 4$ , find the Lagrange interpolation polynomial of degree at most three using the nodes  $x_0 = -3, x_1 = -1, x_2 = 1, x_3 = 5$

Problem 5. Let  $x_0 = -1, x_1 = 0, x_2 = 1$ , define  $f_0(x) = x^2 - 1, f_1(x) = 2x^2 + 3x, f_2(x) = -x^2 + 2x$ . Evaluate these polynomials at  $x_1$ . Uses this to find a polynomial of degree at most 2 such that  $g(x_0) = -4, g(x_1) = -1$ , and  $g(x_2) = 6$  without performing any tedious computations.

1) Task: Taylor expand @  $x = p_n$

$$f(x) = f(p_n) + f'(p_n)(x-p_n) + \frac{f''(\xi)(x-p_n)^2}{2}$$

@  $x = p$

$$0 = f(p) = f(p_n) + f'(p_n)(p-p_n) + \frac{f''(\xi)(p-p_n)^2}{2}$$

$$0 = \frac{f(p_n)}{f'(p_n)} + (p-p_n) + \frac{f''(\xi)(p-p_n)^2}{2f'(p_n)}$$

$$p_{n+1} := p_n - \frac{f(p_n)}{f'(p_n)}$$

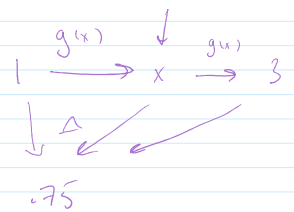
$$0 = p - p_{n+1} + \frac{f''(\xi)(p-p_n)^2}{2f'(p_n)}$$

is  $x$  or  $1.5$

2)  $.75 := \Delta^2(p_0^{(1)}, p_1^{(1)}, p_2^{(1)})$

$$= 1 - \frac{(x-1)^2}{3-2x+1}$$

Solve for  $x$ ,  $x = \{0, 1.5\}$



3) 
$$\begin{array}{cccc|c} 7 & 0 & -2 & -5 & -3 \\ & 7 & 7 & 5 & 0 \\ 7 & 7 & 5 & 0 & -3 = P(1) \end{array}$$

$$P(x) = (7x^3 + 7x^2 + 5x)(x-1) - 3$$

$$(x-1) \overline{7x^4 - 2x^2 - 5x - 3}$$

1

	$x_0$	$x_1$	$x_2$
$f_0$	0	-1	0
$f_1$	-1	0	5
$f_2$	-5	0	1
$g$	2	-1	6

$g$

$$g(x) = f_0 + f_1 + f_2$$

$$g(x) = a f_0 + b f_1 + c f_2$$

$$\begin{cases} 2 = g(x_0) = 0 \cdot a - 1 \cdot b - 3 \cdot c \\ -1 = -a \\ 6 = 5b + c \end{cases}$$

$$\begin{bmatrix} 0 & -1 & -3 \\ -1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

~~1.5~~  
~~0~~ 1.5

If you are given  $x_0, \dots, x_n$

Get  $L_{n,0}, \dots, L_{n,n}$  these are polynomials of degree  $n$

If  $x_i$ 's are distinct, they are a basis of polynomials of degree  $n$

If we want a polynomial  $f$  that interpolates on  $x_i$  then we know that  $f(x) = \sum_{k=0}^n c_k L_{n,k}(x)$

This is true for any basis. But  $L_{n,i}$  are good, because the  $c_k$  are very easy to determine.

They are just  $c_k = f(x_k)$